

$$\begin{aligned} \left(a - \frac{1}{a}\right)^4 &= 9 \\ &= \left(a^2 + \frac{1}{a^2} - 2\right) \left(a^2 + \frac{1}{a^2} - 2\right) \\ &= a^4 + \frac{1}{a^4} - 4a^2 - \frac{4}{a^4} + 6 \\ &= a^4 + \frac{1}{a^4} - 4\left(a - \frac{1}{a}\right)^2 - 2 \\ &= a^4 + \frac{1}{a^4} - 14 = 9 \end{aligned}$$

donc  $a^4 + \frac{1}{a^4} = 23$